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The evolution of star formation efficiency and mass spectrum during the formation of star cluster

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Abstract : The time variation of the star formation efficiency (SFE) is calculated during the formation of star cluster from giant molecular gas cloud. The final form of the mass spectrum of a cluster is also found at the time of gas dispersal for different values of the parameters. It is found that under certain parametric conditions, a cloud can form a bound star cluster and the low mass fragments form a significant fraction of the total stellar mass at the time of gas removal.

Keywords : Star formation efficiency, mass spectrum, dark matter

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1. Introduction

The understanding of the formation process of bound cluster from molecular cloud and the form of the mass spectrum as it evolves is important for investigation of the possible form as well as the amount of baryonic dark matter in the Universe. In recent observations of some galactic globular clusters [1–3], it has been observed that the luminosity function of these clusters (*viz.* NGC 6397, [1]) extends to masses within a few hundredths of a solar mass near the end of the hydrogen burning sequence, and the star counts per unit mass are still rising at this faint limit. In the absence of an identifiable physical cause for a cut-off, it seems more plausible that the mass spectrum which is a power law of the form $n(m) \propto m^{-(1+x)}$ continues into the brown-dwarf regime and with a mass spectral index of $x = 1.0$, brown-dwarfs may constitute half of the total mass of NGC6397. Lada and his coworkers [4] simulated the dynamical evolution of young clusters as they emerge from molecular clouds. They introduced two important parameters in this connection, *viz.*, the star formation efficiency (SFE) *i.e.* the ratio of stellar mass to the total mass including gas and stars, and the gas dispersal time. They found that either the duration of gas dispersal is longer than a million years or the SFE at the time of gas removal is at least 50 percent.

Wilking and Lada [5] observed the central region of the ρ-Ophiuchi cloud and calculated a SFE for its central core to be between 34 to 47 percent. It is shown that if the SFE exceeds 50 percent before the molecular gas is disrupted and dispersed then gravitationally bound cluster will emerge. The fragmentation of a protoglobular cluster cloud (PGCC) has been discussed by Murray and Lin [6,7]. They examined the fragmentation of a PGCC and found that the fragmentation could proceed in two steps. In the first step, thermal instability can lead to the formation of cold, dense, shells in the cloud on a cooling time scale which is much shorter than typical dynamical time scale of the cloud. They investigated the late stages of fragmentation by examining the growth of gravitational instabilities in the shell and found that the short dynamical time scale of the shell leads to the rapid growth of gravitational instabilities so that the entire process of fragmentation to stellar mass scales can be completed on time scales much shorter than the dynamical time scale of the parent cloud. In a more recent work, Kumai and his coworkers [8] discussed how under favourable conditions, a PGCC can give birth to a bound globular cluster with rapid formation of the cluster core heavily populated by low mass stars. Before the gas dispersal by occurrence of supernovae explosion a bound cluster with SFE > 0.5 can form the PGCC under such conditions.

In a previous paper, we had [9] considered the gravitational collapse of a spherically symmetric gas cloud and computed the form of the mass spectrum and the minimum mass of a fragment as the gravitational collapse proceeds. In the present paper, we have considered the evolution of SFE due to coalescence and disintegration among these fragments, as well as the accretion of the residual gas on these fragments during the period after which the gas dispersal occurs. We have computed the evolution of the SFE as well as the form of the mass spectrum during the same period. For this computation, we have used the mass spectral index and the minimum mass of a fragment as obtained in our earlier work [9] as the initial inputs. In Section 2, the mathematical model has been developed and in Section 3, numerical values of the parameters have been discussed. Section 4 gives the computational results while relevant conclusions are drawn in Section 5.

2. Mathematical model

In our earlier paper [9], we have considered the gravitational collapse of a spherically symmetric molecular cloud at low temperature ($T < 500$ K) where molecular hydrogen and dust grains are the only cooling and opacity sources. There we computed the various forms of the initial mass functions (IMF) with a cut-off at minimum mass (M_{min}) for different values of the grain parameters and grain abundances.

Murray and Lin [6,7] have shown that thermal instability in a PGCC is comparable to the cooling time scale τ_c

where $$\tau_c = \frac{3}{2}\frac{kT}{n\Lambda(x,t)}. \tag{2.1}$$

For a cloud of mass $1.6 \times 10^6 M_\odot$, density $n = 270\ \mathrm{cm}^{-3}$, $\tau_c = 0.9\ \tau_d$ *i.e.* $\tau_c < \tau_d$. They found for such a cloud that after 1.58×10^{13} sec ($\sim\tau_c$), density fluctuation $\delta\rho/\rho$ has increased by less than a factor of 2 and this leads to the formation of a cold dense shell. The time scale of gravitational instability τ_g of this cold dense shell is less than τ_c if (ρ/m_H) shell $\geq 1321\ \mathrm{cm}^{-3}$, so that the entire process of fragmentation occurs in a time scale which is shorter than the dynamical time scale *i.e.* $\tau_c + \tau_g \leq \tau_d$. Nakano [10] has shown that the ratio of the collision time t_{col} and dynamical time t_f is

$$t_{col}/t_f \simeq (\pi f)^{-1}(m/M)^{1/3}(\rho/\rho_b)^{7/6}. \tag{2.2}$$

$M = 10^4 M_\odot$, $m = M_\odot$, $\rho/\rho_b = 10$, $f = 0.1$, we obtain $t_{col}/t_f \simeq 3$.

In our calculations, we have considered the gravitational collapse of a molecular cloud whose number density is of the order of $10^4\ \mathrm{cm}^{-3}$, so the dynamical time in that case is of the order of 10^5 years. On the basis of the above discussion, we have considered the collapse in two phases. (i) The fragmentation phase and (ii) the collision disintegration phase. During the former phase, there is only fragmentation *i.e.* the interval for that phase is comparable to the dynamical time *i.e.* the entire process of fragmentation completes within 10^5 years. In the latter case, there is no further fragmentation of the protostars but only inelastic collisions occur among them with the accretion of the residual diffuse gas. As a result of such collisions and accretion, the mass-spectrum of the system changes and the total mass of the proto-stellar part increases, *i.e.* the star formation efficiency (SFE), which is the ratio of the total proto-stellar mass to the total mass of the parent cloud, increases with time. The fragmentation procedure has been discussed in details in our earlier work [9].

Here, we have studied the latter phase only, *i.e.* the collisional phase starting from 10^5 years onwards.

With these points in view, we construct an appropriate model including coalescence, disintegration and accretion. For simplicity, we consider that the collision of fragments is completely inelastic leading to the coalescence of collision partners. We also consider binary collisions only. We assume an initial value of the SFE, $\varepsilon(t_0)$ at the initial time t_0 (= 10^5 years) and study the evolution of the SFE, that is $\varepsilon(t)$, at any subsequent time.

Since we have considered the entire process of fragmentation within 10^5 years, no gas dispersal is important during this period. But as the proto-stars evolve, most of the gas will be ionized and dispersed by the time supernovae begin to explode in the cluster. This latter timescale is of the order of 10^7 years [8,11].

So we assume here that the gas dispersal occurs after 10^7 years and study the evolution of SFE [$\varepsilon(t)$] upto 10^7 years. Thus, 10^7 years has been taken as the time after which the cluster emerges from the parent molecular gas cloud. The gas dispersal which occurs after 10^7 years and onwards, has not been considered in the present model. We have also assumed that in the absence of accretion the total stellar mass is conserved, *i.e.* SFE evolves by accretion only. Coalescence and disintegration have effects only on the nature of stellar mass spectrum, but not on SFE.

Let $N(m, t)$, the number density of fragment of mass m, is increased by coalescence of fragments of masses m^1 and $m - m^1$ and decreased by coalescence of fragment of mass m with others. Let $A_c(m)$ be the rate of accretion on a fragment of mass m. Then the number density $N(m, t)$ is increased by the coalescence of fragment of masses m^1 and $m-m^1$ to form a fragment of mass m which has been represented by the first term on the R.H.S. of equation (2.3) and the number density $N(m, t)$ is decreased by the coalescence of fragments of mass m with a fragment of any other mass which has been represented by the second term with a negative sign. Hence,

$$\frac{\partial N(m,t)}{\partial t}+\frac{\partial}{\partial m}\left[A_c(m)N(m,t)\right]=\frac{1}{2}\int_{M_{min}}^{m-M_{min}}\alpha(m^1,m-m^1)N(m-m^1,t)N(m^1,t)dm^1$$

$$-N(m,t)\int_{M_{min}}^{M_{max}}\alpha(m,m^1)N(m^1,t)dm^1, \qquad (2.3)$$

where $\alpha(m, m^1)$ is the collision parameter between masses m and m^1 and M_{max} is the maximum stellar mass in the cluster. We take $M_{max} = 10\, M_\odot$, since supernovae explosions occur around this mass limit [12].

3. Numerical values of the parameters

3.1. Initial value of the star formation efficiency :

In our earlier work [9], the number density per unit mass has been computed by thus at discrete mass points in the mass range $M_{min} \le m \le 1.2\, M_\odot$ and a power law of the form $N(m, t) = Am^{-\alpha}$ was fitted for the same range. Since in the present calculation, we have used a maximum value of m *i.e.* $M_{max} = 10\, M_\odot$, we have computed the numerical values of the mass function at discrete mass points by the same procedure as done in the earlier paper but extended the range upto a maximum of $10\, M_\odot$. For simplicity, the power law fitted is a single power law instead of a segmented power law and for the particular case of $Z = 10^{-2}$, the value of α fitted is $\alpha = \alpha(t_0) = 2.7$. For $Z = 10^{-2}$, the nature of dust grain used is Graphite and the minimum value of the fragment found is $M_{min} = 0.0033\, M_\odot$. So in our model, we have used the initial form of the mass spectrum as $N(m,t_0) = A(t_0)m^{-\alpha(t_0)}$, for $0.0033\, M_\odot \le m \le 10\, M_\odot$, so that for a core radius $l = 0.1$ pc, the total mass of proto-stars in the cloud is

$$M_*(t_0) = \frac{4}{3}\pi l^3\int_{M_{min}}^{M_{max}}A(t_0)m.m^{-\alpha(t_0)}dm$$

$$= 5.82 \times 10^3\, M_\odot \qquad (3.1.1)$$

Observations of the ρ-Ophiuchi cloud which is at present the best observed example of a proto-cluster system, indicate that the SFE at present may be as high as 30 per cent [5]. Also, Lada and his coworkers [4] suggest by model calculations that this cloud may ultimately form pleiades like cluster which is a galactic cluster. Since the age of the ρ-Ophiuchi cloud is 2×10^6 years *i.e.* older than the just fragmented cloud (age, 10^5 years), in our model we have taken a range of ε (t_0) from 10–30 percent. The values of ε (t_0) used in our model are ε (t_0) = 0.1, 0.2 and 0.3 respectively. Then the total mass of the parent cloud is

$$M_T = M_*(t_0) / \varepsilon (t_0). \tag{3.1.2.}$$

3.2. *The form of impact parameter :*

Field and Saslaw [13] have used the form of α (m, m^1) for cloud-cloud collision as

$$\alpha(m,m^1) = 2\sqrt{2}\pi r^2 v \frac{m+m^1}{(mm^1)^{1/2}}, \tag{3.2.1}$$

if the clouds have same density ($\sigma \propto m^{2/3}$) and are in a condition of equipartition of kinetic energy ($V \propto m^{-1/2}$). We use this expression in our calculations for $m = m^1$. Then α (m, m) = $4\pi r^2 v\sqrt{2}$ where v, r are the mean speed and radius of the fragment of mass m. Since the mass spectrum at initial time $t_0 = 10^5$ years is very steep with a value of the exponent α (t_0) = 2.7, the low mass fragments contribute a significant part (almost as high as 90 percent) to the total proto-stellar mass. Hence, we assume that the collision among the brown dwarfs (mass ~0.08 $M_\odot$) is dominant. We take $V = \langle v^2 \rangle^{1/2}$ for the velocity of the brown-dwarfs. Now $\langle v^2 \rangle^{1/2} = \sqrt{\frac{GM_T}{R}}$ where M_T is the total mass of the parent cloud, R being the size of the cloud. Since the size of the molecular clouds vary from 10–20 pc, we take R = 10 pc in our case and that gives v ~5 km s^{-1}.

Also for masses of the order of brown dwarfs, the average density is of the order of 10^{-13} g cm^{-3} [14]. For the above estimates of mass and density, the radius is of the order of 10^4 R. Since the density is very low, the collapse initially is of free fall type until it reaches the quasi-hydrostatic equilibrium phase (QHE) when the collapse becomes very slow. It can be shown that proto-stars having radii ~10^5 R., decrease to the radii by two order of magnitudes within a very short time (~10^3 yrs.) while it switches over to QHE phase [15]. So in our model, we have taken the radius of the brown dwarf contracts to 10^2 $R_\odot$ after that short period of time and since the collapse is then quasi-hydrostatic, the contraction is not remarkable. Moreover at 10^6 years, deuterium burning starts and the radius is then almost constant during 10^6 – 10^7 years [16]. So within the entire period of our calculation, the radius of the brown dwarf does not deviate much from the value of 10^2 $R_\odot$. After 10^7 years when deuterium burning is completed, the brown dwarf will slowly but continuously contract for its evolution because its only source of luminosity is the contraction energy, there being no nuclear burning at the centre. It is very likely therefore, that at the age of a Gyr, the radius of the brown dwarf will reduce to a very small size.

3.3. The form of accretion rate :

Spitzer [17] considered the accretion by a massive object surrounded by extended gas cloud. The rate of accretion is

$$\frac{dM}{dt} \qquad 4\pi(GM)^2\rho_1 \tag{3.3.1}$$

where M is the mass of the object and ρ_1 and v are respectively the density and velocity of the accreting material at infinity. In a molecular cloud, the density varies from 10^4 to 10^5 cm^{-3} from outer surface towards the core [18,19]. So we consider the number density from 5×10^4 cm^{-3} to 10^5 cm^{-3} for ρ_1/m_H and have taken v to be 5 kms^{-1} in our calculation. With these values, the rate of accretion for $M = 1\ M_\odot$ varies from 4×10^{-9} to $9 \times 10^{-9}\ M_\odot\ yr^{-1}$ as follows from eq. (3.3.1).

In the above calculation, Spitzer has assumed that the accreting material has infinite mean free path; but in practice, the accreting material will undergo frictional resistance which slows down the rate of accretion. So instead of the above expression in (3.3.1), we have adopted the rate given by [20]

$$\frac{dM}{dt} = Bm^{(1-p)/3}, \tag{3.3.2}$$

where B and p are constants. For inelastic collision, $\frac{d^2M}{dt^2} > 0$ which gives $p < 1$ and since $p > 0$, so $0 < p < 1$. If $\gamma = \frac{1-p}{3}$ then $0 < \gamma < 0.33$.

The rate of accretion is $\frac{dM}{dt} = Bm^{\gamma}$, $o < \gamma < 0.33$. (3.3.3).

In our model, we have assumed the accretion rate in such a way that the rate is normalised with the above one in (3.3.1) at $m = 1\ M_\odot$, which gives $4 \times 10^{-9} \leq B \leq 9 \times 10^{-9}$.

4. Computational results

For numerical calculations, we have solved the integro-differential equation (2.3) for the values of $N\ (m,\ t)$ for different values of m from $M_{\min}$ to $M_{\max}$ at any arbitrary time t and fitted the discrete points by a power law of the form

$$N(m,t) = A(t)m^{-\alpha(t)}, \qquad M_{\min} \leq m \leq M_{\max}, \tag{4.1}$$

where $A\ (t)$, $\alpha\ (t)$ are functions of time only. Then at $t = 10^7$ years, the total stellar mass is

$$M_*(t) = \frac{4}{3}\pi l^3 \int_{M_{\min}}^{M_{\max}} A(t)m.m^{-\alpha(t)}dm, \tag{4.2}$$

where l is the core radius of the cluster. If, M_T is the total mass of the parent cloud then

$$\varepsilon\ (t) = M_*\ (t)/M_T.$$

The values of $\varepsilon(t)$ and $\alpha(t)$ at $t = 10^7$ years are given in Table 1 (columns 4 and 5) for different values of the parameters. Also the percentage ratio of total mass of proto-stellar fragments in the brown dwarf regime ($M_{min} \leq m \leq 0.08\ M_{\odot}$) to the total stellar mass at $t = 10^7$ years, are calculated and shown in Table 1 (column 6).

Table 1. The values of the efficiencies ($\varepsilon(t)$) and mass spectral indices ($\alpha(t)$) of a star-cluster at the time of gas dispersal for different values of the accretion rates and initial efficiencies with $\alpha(t_0) = 2.7$, $< v^2 >^{1/2} = 5\ \mathrm{kms}^{-1}$.

$\varepsilon(t_0)$ (%)		$B/10^{-9}$	$\varepsilon(t)$ (%)	$\alpha(t)$	M^1_{BD}
10	0.1	4	36.44	2.03	43
		5	46.26	1.98	38
		6	56.88	1.94	34
		7	68.20	1.91	31
		8	80.12	1.88	29
		9	92.50	1.86	27
10	0.2	4	25.33	2.09	48
		5	30.73	2.05	45
		6	36.65	2.02	42
		7	43.03	1.99	39
		8	49.82	1.96	36
		9	56.96	1.94	34
20	0.3	4	37.92	2.14	53
		5	43.86	2.11	50
		6	50.35	2.09	48
		7	57.38	2.06	46
		8	64.89	2.04	45
		9	72.86	2.02	42
30	0.3	4	53.18	2.05	45
		5	60.68	2.03	43
		6	68.87	2.07	42
		7	77.70	2.00	40
		8	87.16	1.98	38
		9	97.19	1.97	37

It is clear from Table 1 that when the initial efficiency ($\epsilon(t_0)$) for star formation is low (*viz*. 10 percent), the efficiency for the formation of a bound cluster *i.e.* $\epsilon(t) \geq 0.5$ is favoured for smaller values of γ (*viz*, $\gamma = 0.1, 0.2$) and when the initial efficiency is comparatively higher (*viz*. $\epsilon(t_0) = 0.2, 0.3$) the formation of a bound cluster is favoured at somewhat higher value (*viz*. $\gamma = 0.3$) of γ.

This is because when the values of both $\epsilon(t_0)$ and γ are low, the initial value of the effective accretion index $(\gamma - \alpha)$ is high in comparison with that when both of them are high. So the building up of the efficiency is rather fast in the former case resulting in a high value of SFE at 10^7 years.

The values of $\alpha(t)$ (*viz* Figure 1) are decreasing from their initial values ($\alpha(t_0)$) as the cluster emerges from its parent molecular cloud. The inelastic collision between the stars in the system gradually depletes the number of low mass stars causing a decrease in the steepness of the mass-spectrum.

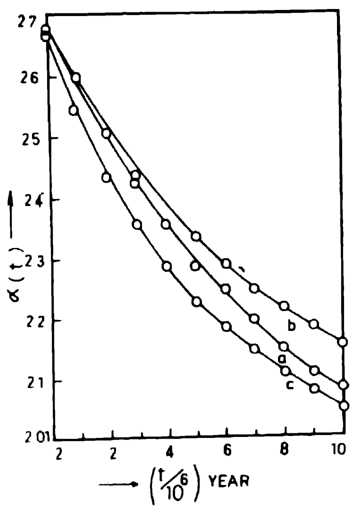

Figure 1. Variation of $\alpha(t)$ with time for ·
a) $\varepsilon = 10\%$, $K = 4 \times 10^{-9}$, $\gamma = 2$
b) $\varepsilon = 20\%$, $K = 4 \times 10^{-9}$, $\gamma = 3$
c) $\varepsilon = 30\%$, $K = 4 \times 10^{-9}$, $\gamma = .3$

In our present computation, α at $t = 10^7$ years (Table 1, column 5) varies from 1.86 to 2.16, *i.e.* α ~2.0 which is consistent with the recent work carried out by Burrows and his coworkers [21]. In their paper, they have computed the evolution of the theoretical mass functions from 10^6 to 2×10^{10} years in the mass range 0.01 $M_\odot$ to 0.2 $M_\odot$ for $Z = Z_\odot$, deuterium fraction $Y_d = 2 \times 10^{-5}$, Helium fraction $Y_\alpha = 0.25$ and mixing length parameter $\alpha = l_{p/H} = 1$. They find that deuterium burning continues upto 10^7 years and then luminosities of the stars and proto-stars continue to decay. Subsequently near 2×10^8 years, hydrogen burning commences for massive ($> 0.1\ M_\odot$) objects and near 10^9 years for objects having masses ~$0.07 M_\odot$, while for substellar objects ($M < 0.0767 M_\odot$), luminosity continues to decay. So there is a segregation in the luminosity of order six between very low mass stars and substellar objects within 10^8 – 10^9 years (Figure 3, [21]) Then they derived the luminosity function between 10^6 – 2×10^{10} years *via* the assumed mass formula

$$\xi(M)dM = CM^{-\alpha}dM,$$

taking α as a parameter. Finally, the theoretical mass functions were fitted with the young ρ-Ophiuchi cluster (age = 1.5×10^6 years) at 1.5×10^6 years and the old Hyades cluster (age = 6×10^9 years) at 6×10^9 years. For ρ-Ophiuchi cloud, the fit was most satisfactory for $\alpha = 2.35$ whereas for Hyades, it was for $\alpha = 0$ to 1.4. So it is clear from the above results that initially ($t < 10^7$ years) so far as the brown dwarfs remain as deuterium burners, they contribute a significant part of the luminosity function in the low-mass region and so the observed mass spectrum remains steeper in this region (*e.g.* Salpeter mass-spectrum, $\alpha = 2.35$); but as the cluster ages the, brown-dwarf luminosity decays significantly and so the observed luminosity function becomes flatter in the low-mass region and the consequent IMF in this region will appear to be flatter as has been suggested by some authors. The observed value of α in the former case is therefore, consistent more or less with our computed result ($\alpha \sim 2.0$). Theoretical luminosity functions were also derived at various ages in the mass range $0.03\ M_\odot < M < 0.2\ M_\odot$ by Hubbard and his coworkers [22] and compared with that of Hyades at $t = 6 \times 10^9$ yrs taking α as a parameter but for their study, the satisfactory fit was for $\alpha = 0$ than for Salpeter spectrum ($\alpha = 2.35$). However their result has been contradicted by Henry [23] who found that $\alpha = 0$ for Hyades is inconsistent with his value in the solar neighbourhood. With the new data of Bryja and his coworkers [24] and a redetermination of the Hyades index as being between roughly 0 and 1, the Hyades and the solar neighbourhood results are now consistent. Several observations on the luminosity function also indicate a significant contribution from the end of brown dwarfs which are consistent with our calculations (Table 1, column 6), *e.g.* the construction of the infrared observed luminosity functions for South Galactic Pole as well as for Hyades cluster by Leggett and Hawkins [25]. The luminosity function in each case shows a peak at $M = 0.25\ M_\odot$ and then decreases at the minimum with masses $\sim 0.1\ M_\odot$ and it shows evidence for a subsequent rise for extremely low mass objects. Similar luminosity function at the end of the main sequence has been determined from V, R and I data by CCD/Transit Instrument (*CTI*), covering an area of 2.7 deg^2 from $b = +90°$ to $-35°$ by Kirpatrik and his coworkers [26]. The construction of mass function in the core of ρ-Ophiuchi cloud from its luminosity function was carried out by Comeron and his coworkers [27]. The luminosity function in their model was based on fitting isochrones from theoretical models. The mass function estimated is of the form

$$n(L)\, dL = AL^b\, dL,$$

where $n(L)$ is the number density of stars with luminosities between L and $L + dL$. They had a best fit for $b = -1.21$ and the IMF extends to well below the bottom of the main sequence ($0.08\ M_\odot$). Though the slope is rather flat compared to our result (~ -2.0) still it gives an observational support to some extent that the brown dwarfs form in significant numbers during the fragmentation of molecular clouds and the process of cloud fragmentation is not biased against substellar masses. Our present computation shows a strong correlation with the mass spectrums observed for seven globular clusters by Fahlman and coworkers [1] and Richer *et al* [28]. They studied seven globular clusters having their main sequences extending down to near the limit. None of these clusters show any evidence of a flattening or turnover in slope at the lowest masses, some may have IMF slopes steeper than the Salpeter value long associated with the galactic disk. Richer and Fahlman [29] have also estimated a corresponding IMF for the halo population. The IMF rises faster than a Salpeter slope down to 0.14 $M_\odot$. Extension of this slope down to 0.01 $M_\odot$ would allow VLM's and brown dwarfs to dominate the dark matter content of the galaxy. The expected number density of halo brown-dwarfs in the solar neighbourhood would be 0.5 pc^{-3}. Also a near infrared study were carried out by Simons and Becklin [30] in Pleiades cluster in search of brown-dwarfs. They detected 22 ± 10 objects which lie near lower Pleiades main sequence. These objects have masses in the range of ~ 0.1 – 0.04 $M_\odot$ and the power law fitted to these masses has the slope of – 2.8 ± 0.5, which is more or less compatible to our result. This study therefore, indicates that associated with every bound cluster, there is likely to be a significant amount of mass in brown-dwarfs.

5. Conclusions

(i) After the entire process of fragmentation of an isothermally and spherically collapsing molecular cloud, the mass spectrum is a power law of the form $n(m) \alpha\, m^{-\alpha(t_0)}$ [9] and subsequently the fragments evolve by nutual collisions as well as accretion of the residual gas. The parent cloud can produce a bound cluster *i.e.* the SFE at the time of gas removal, exceeds 50 percent and can attain a much higher value under most favourable conditions.

(ii) The formation of a bound cluster from its parent cloud is favoured both in the case of low initial efficiency and low value of γ as well as in the case of high initial efficiency and comparatively higher value of γ.

(iii) The mass spectrum becomes more flat than its initial form.

(iv) The mass spectrum at the time of gas dispersal is consistent with the observation of the mass spectra of various star clusters.

(v) Sufficient unseen matter in the form of brown-dwarfs is likely to exist within a star-cluster.

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